Учебен център “СОЛЕМА”
обучение по математика, физика, български и английски език, компютър
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Тригонометрични формули

(навсякъде във формулите “+” или “– “ пред корена имаме в зависимост на това в кой квадрант се намира второто рамо на ъгъла)

1. Основни тригонометрични равенства:
   - 1.1: $\sin^2 \alpha + \cos^2 \alpha = 1$
   - 1.2: $tg\, \alpha\, cotg\, \alpha = 1$
   - 1.3: $tg\, \alpha = \dfrac{\sin \alpha}{\cos \alpha}$; $\cot g\, \alpha = \dfrac{\cos \alpha}{\sin \alpha}$
2. Връзка между тригонометричните функции:

	sin α	cos α
sin α =	sin α	(2.1): $\pm\sqrt{1-\cos^2\alpha}$
cos α =	(2.2): $\pm\sqrt{1-\sin^2\alpha}$	cos α
tg α =	(2.3): $\dfrac{\sin\alpha}{\pm\sqrt{1-\sin^2\alpha}}$	(2.4): $\dfrac{\pm\sqrt{1-\cos^2\alpha}}{\cos\alpha}$
cotg α =	(2.5): $\dfrac{\pm\sqrt{1-\sin^2\alpha}}{\sin\alpha}$	(2.6): $\dfrac{\cos\alpha}{\pm\sqrt{1-\cos^2\alpha}}$

	tg α и tg α/2	cotg α
sin α =	(2.7): $\dfrac{tg\,\alpha}{\pm\sqrt{1+tg^2\alpha}}$ (2.8): $\dfrac{2tg\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$	(2.9): $\dfrac{1}{\pm\sqrt{1+\cot g^2\alpha}}$
cos α =	(2.10): $\dfrac{1}{\pm\sqrt{1+tg^2\alpha}}$ (2.11): $\dfrac{1-tg^2\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$ (2.12): $\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}$	(2.13): $\dfrac{\cot g\alpha}{\pm\sqrt{1+\cot g^2\alpha}}$
tg α =	(2.14): $\dfrac{2tg\frac{\alpha}{2}}{1-tg^2\frac{\alpha}{2}}$ (виж 16)	(2.15): $\dfrac{1}{\cot g\alpha}$
cotg α =	(2.16): $\dfrac{1}{tg\,\alpha}$	(2.17): $\dfrac{1-tg^2\frac{\alpha}{2}}{2tg\frac{\alpha}{2}}$ (виж 16)

3. Формули за понижаване на степен:
   - 3.1: $2\sin^2\alpha = 1 - \cos 2\alpha$
   - 3.2: $\sin^2\alpha = \dfrac{tg^2\alpha}{1+tg^2\alpha}$
   - 3.3: $2\cos^2\alpha = 1 + \cos 2\alpha$
   - 3.4: $\cos^2\alpha = \dfrac{1}{1+tg^2\alpha}$
   - 3.5: $4\sin^3\alpha = 3\sin\alpha - \sin 3\alpha$
   - 3.6: $4\cos^3\alpha = 3\cos\alpha + \cos 3\alpha$
   - 3.7: $8\sin^4\alpha = \cos 4\alpha - 4\cos 2\alpha + 3$
   - 3.8: $8\cos^4\alpha = \cos 4\alpha + 4\cos 2\alpha + 3$
   - 3.9: $\sin^4\alpha - \cos^4\alpha = \sin^2\alpha - \cos^2\alpha$
   - 3.10: $\cot g^2\alpha = \dfrac{1-\sin 2\alpha}{1+\sin 2\alpha}$
   - 3.11: $2\sin^2\dfrac{\alpha}{2} = 1 - \cos\alpha$
   - 3.12: $2\cos^2\dfrac{\alpha}{2} = 1 + \cos\alpha$
4. Формули за сбор и разлика на два ъгъла:
   - 4.1: $\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
   - 4.2: $\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
   - 4.3: $\cos(\alpha+\beta)\cos(\alpha-\beta) = \cos^2\beta - \sin^2\alpha$
   - 4.4: $\sin(\alpha+\beta)\sin(\alpha-\beta) = \sin^2\alpha - \sin^2\beta$
   - 4.5: $tg(\alpha\pm\beta) = \dfrac{tg\,\alpha \pm tg\,\beta}{1\mp tg\,\alpha\cdot tg\,\beta}$
   - 4.6: $\cot g(\alpha\pm\beta) = \dfrac{\cot g\alpha\cdot\cot g\beta \mp 1}{\cot g\beta \pm \cot g\alpha}$
5. Двойни, тройни и половинки ъгли:
   - 5.1: $\sin 2\alpha = 2\sin\alpha\cos\alpha = (\sin\alpha+\cos\alpha)^2 - 1$ (виж 17)
   - 5.2: $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \cos^4\alpha - \sin^4\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha = \dfrac{1-4\sin^2\alpha\cdot\cos^2\alpha}{\cos^2\alpha-\sin^2\alpha}$ (виж 1) $= \dfrac{1-tg^2\alpha}{1+tg^2\alpha}$ (виж 15)
   - 5.3: $tg\,2\alpha = \dfrac{2tg\,\alpha}{1-tg^2\alpha} = \dfrac{2}{\cot g\alpha - tg\,\alpha}$
   - 5.4: $\cot g\,2\alpha = \dfrac{\cot g^2\alpha - 1}{2\cot g\alpha} = \dfrac{\cot g\alpha - tg\,\alpha}{2}$
   - 5.5: $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha = \sin\alpha\,(3-4\sin^2\alpha) = \cos\alpha\,(2\cos 2\alpha - 1) = \sin\alpha\,(4\cos^2\alpha - 1) = 4\sin\alpha\sin(60^0+\alpha)\sin(60^0-\alpha) = 3\cos^2\alpha\sin\alpha - \sin^3\alpha$ (виж 3)
   - 5.6: $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha = \cos\alpha\,(4\cos^2\alpha - 3) = \cos\alpha\,(1-4\sin^2\alpha) = 4\cos\alpha\sin(30^0-\alpha)\sin(30^0+\alpha) = \cos^2\alpha - 3\cos\alpha\sin^2\alpha$ (виж 2)
   - 5.7: $tg\,3\alpha = tg\,\alpha\; tg\,(600-\alpha)\; tg\,(600+\alpha)$
   - 5.8: $tg\,3\alpha = \dfrac{3tg\,\alpha - tg^3\alpha}{1-3tg^2\alpha}$
   - 5.9: $cotg\,3\alpha = cotg\,\alpha\; cotg\,(60^0-\alpha)\; cotg\,(60^0+\alpha)$
   - 5.10: $\cot g\,3\alpha = \dfrac{\cot g^3\alpha - 3\cot g\alpha}{3\cot g^2\alpha - 1}$
   - 5.11: $\cos 4\alpha = \cos^4\alpha - 6\cos^2\alpha\sin^2\alpha + \sin^4\alpha$ (виж 2)
   - 5.12: $\sin 4\alpha = 4\cos^3\alpha\sin\alpha - 4\cos\alpha\sin^3\alpha$ (виж 3)
   - 5.13: $\sin\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$

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5.14: $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

5.15: $tg\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}$

5.16: $\cot g\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}} = \frac{\sin\alpha}{1-\cos\alpha} = \frac{1+\cos\alpha}{\sin\alpha}$

5.17: $tg(\frac{\pi}{4}+\alpha) = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha}$

6. Преобразуване на произведение в алгебричен сбор:

6.1: $\sin\alpha\cdot\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$

6.2: $\cos\alpha\cdot\cos\beta = \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$

6.3: $\sin\alpha\cdot\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$

6.4: $tg\alpha\cdot tg\beta = \frac{tg\alpha+tg\beta}{\cot g\alpha+\cot\beta} = -\frac{tg\alpha-tg\beta}{\cot g\alpha-\cot g\beta}$

6.5: $\cot g\alpha\cdot\cot g\beta = \frac{corg\alpha+\cot g\beta}{tg\alpha+tg\beta} = -\frac{\cot g\alpha-\cot g\beta}{tg\alpha-tg\beta}$

6.6: $\cot g\alpha\cdot tg\beta = \frac{\cot g\alpha+tg\beta}{tg\alpha+\cot g\beta} = -\frac{\cot g\alpha-tg\beta}{tg\alpha-\cot g\beta}$

7. Преобразуване на сбор или разлика в произведение:

7.1: $\sin\alpha+\sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

7.2: $\sin\alpha-\sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$

7.3: $\cos\alpha+\cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

7.4: $\cos\alpha-\cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

7.5: $tg\alpha\pm tg\beta = \frac{\sin(\alpha\pm\beta)}{\cos\alpha\cos\beta}$

7.6: $\cot g\alpha\pm\cot g\beta = \frac{\sin(\beta\pm\alpha)}{\sin\alpha\sin\beta}$

8. Преобразуване във вид удобен за логаритмуване:

8.1: $1+\sin\alpha = 1+\cos(90^0-\alpha) = 2\cos^2\left(45^0-\frac{\alpha}{2}\right)$

$= \left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right)^2$ (*виж* 14)

8.2: $1-\sin\alpha = 2\sin^2\left(45^0-\frac{\alpha}{2}\right)$

8.3: $1\pm tg\alpha = \frac{\sin(45^0\pm\alpha)}{\cos 45^0\cos\alpha} = \frac{\sqrt{2}\sin(45^0\pm\alpha)}{\cos\alpha}$ (виж 4.)

8.4: $1\pm tg\alpha tg\beta = \frac{\cos(\alpha\mp\beta)}{\cos\alpha\cos\beta}$

8.5: $\cot g\alpha\cot g\beta\pm 1 = \frac{\cos(\alpha\mp\beta)}{\sin\alpha\sin\beta}$

8.6: $1-tg^2\alpha = \frac{\cos 2\alpha}{\cos^2\alpha}$ (виж 5)

8.7: $1-\cot g^2\alpha = -\frac{\cos 2\alpha}{\sin^2\alpha}$

8.8: $A+B\sin\alpha = 2B\sin\frac{\alpha+\varphi}{2}\cos\frac{\alpha-\varphi}{2}, ако \left|\frac{A}{B}\right|\le 1, \sin\varphi = \frac{A}{B}$

$A-B\sin\alpha = 2B\sin\frac{\alpha-\varphi}{2}\cos\frac{\alpha+\varphi}{2}$

8.9: $C+D\cos\alpha = 2D\cos\frac{\alpha+\theta}{2}\cos\frac{\alpha-\theta}{2}, ако \left|\frac{C}{D}\right|\le 1, \cos\theta = \frac{C}{D}$

$E-F\cos = -2F\sin\frac{\alpha+\theta}{2}\sin\frac{\alpha-\theta}{2}, ако \left|\frac{E}{F}\right|\le 1, \cos\theta = \frac{E}{F}$

8.10: $\sin\alpha+\sin 2\alpha+\ldots+\sin n\alpha = \frac{\cos\frac{\alpha}{2}-\cos\frac{(2n+1)\alpha}{2}}{2\sin\frac{\alpha}{2}}$

8.11: $\cos\alpha+\cos 2\alpha+\ldots+\cos n\alpha = \frac{\sin\frac{(2n+1)\alpha}{2}-\sin\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}}$

8.12: $\sin\alpha+\cos\alpha = \sqrt{2}\cos(45^0-\alpha)$ (виж 7)

$\sin\alpha+\cos\alpha = \sqrt{2}\sin(\alpha+45^0)$

8.13: $\cos\alpha-\sin\alpha = \sqrt{2}\sin(45^0-\alpha)$

$\sin\alpha-\cos\alpha = \sqrt{2}\sin(\alpha-45^0)$

8.14: $tg\alpha+\cot g\beta = \frac{\cos(\alpha-\beta)}{\cos\alpha\sin\beta}; tg\alpha-\cot g\beta = -\frac{\cos(\alpha+\beta)}{\cos\alpha\sin\beta}$

8.15: $1+\sin\alpha+\cos\alpha = 2\sqrt{2}\cos\frac{\alpha}{2}\sin\left(\frac{\alpha}{2}+45^0\right)$ (виж 6)

8.16: $\sin\alpha+\sqrt{3}\cos\alpha = 2\sin(\alpha+60^0)$ (виж 8)

8.17: $\sin\alpha+\cos 3\alpha = 2\sin(45^0-\alpha)\cos(2\alpha-45^0)$(виж 9)

8.18: $3\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2} = 4\cos\left(30^0+\frac{\alpha}{2}\right)\cos\left(30^0-\frac{\alpha}{2}\right)$ (виж 10)

8.19: $4\sin^2\frac{\alpha}{2}-1 = 4\sin\left(\frac{\alpha}{2}+30^0\right)\sin\left(\frac{\alpha}{2}-30^0\right)$ (виж 11)

8.20: $3-4\sin^2\alpha = \frac{\sin 3\alpha}{\sin\alpha}$ (виж 13)

8.21: $3-tg^2\alpha = \frac{\sin 3\alpha}{\sin\alpha\cos^2\alpha}$

8.22: $\cot g^2\alpha-3 = \frac{\cos 3\alpha}{\cos\alpha\sin^2\alpha}$

8.23: $\sin^2\alpha-\sin^2\beta = \cos^2\beta-\cos^2\alpha = \sin(\alpha+\beta)\sin(\alpha-\beta)$

8.24: $\cos^2\alpha-\sin^2\beta = \cos^2\beta-\sin^2\alpha = \cos(\alpha+\beta)\cos(\alpha-\beta)$

8.25: $tg^2\alpha-tg^2\beta = \frac{\sin(\alpha+\beta)\sin(\beta-\alpha)}{\cos^2\alpha\cos^2\beta}$

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8.26: $\cot g^2\alpha - \cot g^2\beta = \dfrac{\sin(\alpha+\beta)\sin(\beta-\alpha)}{\sin^2\alpha\sin^2\beta}$

8.27: $tg^2\alpha - \sin^2\alpha = tg^2\alpha\sin^2\alpha$

8.28: $cotg^2\alpha - \cos^2\alpha = cotg^2\alpha\cos^2\alpha$

Преобразувания

1. $\cos 2\alpha = \dfrac{\cos^2 2\alpha}{\cos 2\alpha} = \dfrac{1-1+\cos^2 2\alpha}{\cos^2\alpha - \sin^2\alpha} = \dfrac{1-(1+\cos 2\alpha)(1-\cos 2\alpha)}{\cos^2\alpha-\sin^2\alpha} = \dfrac{1-4\dfrac{1+\cos 2\alpha}{2}\dfrac{1-\cos 2\alpha}{2}}{\cos^2\alpha-\sin^2\alpha} = \dfrac{1-4\cos^2\alpha\sin^2\alpha}{\cos^2\alpha-\sin^2\alpha}$

2. $\cos n\alpha = \cos^n\alpha - C_n^2\cos^{n-2}\alpha\sin^2\alpha + C_n^4\cos^{n-4}\alpha\sin^4\alpha - \ldots$

3. $\sin n\alpha = n\cos^{n-1}\alpha\sin\alpha - C_n^3\cos^{n-3}\alpha\sin^3\alpha + C_n^5\cos^{n-5}\alpha\sin^5\alpha - \ldots$

където $C_n^k = \dfrac{n(n-1)(n-2)\ldots(n-k+1)}{k!} = \dfrac{n(n-1)(n-2)\ldots(n-k+1)}{1.2.3\ldots k}$

4. $1+tg\alpha = tg45^0 + tg\alpha = \dfrac{\sin 45^0}{\cos 45^0} + \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{\sin 45^0\cos\alpha + \cos 45^0\sin\alpha}{\cos 45^0\cos\alpha} = \dfrac{\sin(45^0+\alpha)}{\dfrac{\sqrt{2}}{2}\cos\alpha} = \dfrac{\sqrt{2}\sin(45^0+\alpha)}{\cos\alpha}$

5. $1-tg^2\alpha = 1-\dfrac{\sin^2\alpha}{\cos^2\alpha} = \dfrac{\cos^2\alpha-\sin^2\alpha}{\cos^2\alpha} = \dfrac{\cos 2\alpha}{\cos^2\alpha}$

6. $1+\sin\alpha+\cos\alpha = (1+\cos\alpha)+\sin\alpha = 2\cos^2\dfrac{\alpha}{2} + 2\sin\dfrac{\alpha}{2}\cos\dfrac{\alpha}{2} = 2\cos\dfrac{\alpha}{2}\left(\sin\dfrac{\alpha}{2}+\cos\dfrac{\alpha}{2}\right) = 2\sqrt{2}\cos\dfrac{\alpha}{2}\left(\dfrac{\sqrt{2}}{2}\sin\dfrac{\alpha}{2}+\dfrac{\sqrt{2}}{2}\cos\dfrac{\alpha}{2}\right) =$

$= 2\sqrt{2}\cos\dfrac{\alpha}{2}\left(\sin\dfrac{\alpha}{2}\cos 45^0 + \cos\dfrac{\alpha}{2}\sin 45^0\right) = 2\sqrt{2}\cos\dfrac{\alpha}{2}\sin\left(\dfrac{\alpha}{2}+45^0\right)$

7. $\sin\alpha+\cos\alpha = \sin\alpha + \sin\left(\dfrac{\pi}{2}+\alpha\right) = 2\sin\left(\alpha+\dfrac{\pi}{4}\right)\cos\dfrac{\pi}{4} = 2\sin\left(\alpha+\dfrac{\pi}{4}\right)\dfrac{\sqrt{2}}{2} = \sqrt{2}\sin\left(\alpha+\dfrac{\pi}{4}\right)$

8. $\sin\alpha + \sqrt{3}\cos\alpha = 2\left(\dfrac{1}{2}\sin\alpha + \dfrac{\sqrt{3}}{2}\cos\alpha\right) = 2(\sin\alpha\cos 60^0 + \cos\alpha\sin 60^0) = 2\sin(\alpha+60^0)$

9. $\sin\alpha + \cos 3\alpha = \sin\alpha + \sin(90^0-3\alpha) = 2\sin\dfrac{\alpha+90^0-3\alpha}{2}\cos\dfrac{\alpha-90^0+3\alpha}{2} = 2\sin(45^0-\alpha)\cos(2\alpha-45^0)$

10. $3\cos^2\dfrac{\alpha}{2} - \sin^2\dfrac{\alpha}{2} = 2\cos^2\dfrac{\alpha}{2} + \cos^2\dfrac{\alpha}{2} - \sin^2\dfrac{\alpha}{2} = 1+\cos\alpha+\cos\alpha = 1+2\cos\alpha = 4\cos\left(30^0+\dfrac{\alpha}{2}\right)\cos\left(30^0-\dfrac{\alpha}{2}\right)$

11. $4\sin^2\dfrac{\alpha}{2} - 1 = 2.2\sin^2\dfrac{\alpha}{2} - 1 = 2(1-\cos\alpha) - 1 = 2-2\cos\alpha - 1 = 1-2\cos\alpha = 4\sin\left(\dfrac{\alpha}{2}+30^0\right)\sin\left(\dfrac{\alpha}{2}-30^0\right)$

13. $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha \Leftrightarrow \sin 3\alpha = \sin\alpha(3-4\sin^2\alpha) \Leftrightarrow 3-4\sin^2\alpha = \dfrac{\sin 3\alpha}{\sin\alpha}$

14. $1+\sin\alpha = \sin^2\dfrac{\alpha}{2} + \cos^2\dfrac{\alpha}{2} + \sin 2\dfrac{\alpha}{2} = \sin^2\dfrac{\alpha}{2} + \cos^2\dfrac{\alpha}{2} + 2\cos\dfrac{\alpha}{2}\sin\dfrac{\alpha}{2} = \left(\sin\dfrac{\alpha}{2}+\cos\dfrac{\alpha}{2}\right)^2$

15. $\cos 2x = \cos^2 x - \sin^2 x = \dfrac{\cos^2 - \sin^2 x}{\cos^2 + \sin^2 x} = \dfrac{1-\dfrac{\sin^2 x}{\cos^2 x}}{1+\dfrac{\sin^2 x}{\cos^2 x}} = \dfrac{1-tg^2 x}{1+tg^2 x}$

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16.
$$tgx=\frac{\sin x}{\cos x}=\frac{\frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}}}{\frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}}}=\frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}}$$

17. $(\sin x + \cos x)^2 - 1 = \sin^2 x + 2\sin x.\cos x + \cos^2 x - 1 = 1 + 2\sin x.\cos x - 1 = 2\sin x.\cos x = \sin 2$